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Essential Graduate Physics

Lecture Notes and Problems

2024

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Short Preface

(for all prospective readers)

1. **Contents**: This series consists of four subject parts: [2]

CM: *Classical Mechanics* (for a 1-semester course),
EM: *Classical Electrodynamics* (2 semesters),
QM: *Quantum Mechanics* (2 semesters), and
SM: *Statistical Mechanics* (1 semester),

and also includes two common appendices:

MA: Selected Mathematical Formulas (16 pp.) and
UCA: Selected Units and Constants (4 pp.),

a common list of *References* (2 pp.), and the *Front Matter* you are reading now, including two prefaces, the list of used notation, and the general list of contents.

2. **Problems**: Each chapter of the lecture notes ends with a list of exercise problems. Detailed model solutions of these problems are available for free download from

https://essentialgraduatephysics.org/.

3. **Hard copies**: Besides free online access to all materials of the series, their B/W paperback copies (one lecture notes volume and one problem solutions volume for each subject part, plus a common *Front Matter and Appendices* brochure) are also available on *Amazon.com*:

https://www.amazon.com/stores/page/DAA96D04-4C1B-414C-B3E6-2F1E934A9561

for the lowest prices covering the printing costs and the distributor's charges.

4. **Basic distinctive features** of the series:

—succinct lecture notes: ~200 pp. per semester;
—focus on problem-solving skills: almost 1,200 solved problems;
—extensive cross-referencing between the parts.

5. **Precursors**: Introductory university/college courses on physics and mathematics.[3] (An additional undergraduate-level course on each subject to be studied is a plus but not an absolute must.)

6. **Goal final level**: Sufficient to pass the corresponding course and comprehensive PhD exams in a highly-ranked university.

Enjoy!

[2] Their texts are saved as separate pdf files of each chapter and each appendix, optimized for two-page viewing and double-side printing. Merged files for each part and the series as a whole, which are especially convenient for search purposes, are also provided.
[3] The math preparation of the reader should be sufficient to understand (but not necessarily remember!) the formulas listed in the MA appendix.

Detailed Preface

(for prospective instructors and other thorough readers)

1. Motivation and History

The series is a by-product of the so-called *core physics courses* I taught in the Department of Physics and Astronomy of Stony Brook University from 1991 to 2013. Reportedly, most other physics departments also require their graduate students to either take a set of similar courses, or pass comprehensive exams based on an approximately similar body of knowledge, or both. This is why I hope that my materials may be useful for instructors and students of such courses, as well as for individual learners. All four subject parts of the series share a common teaching style, structure, and notation, and are interlinked by extensive cross-referencing.

The motivation for composing the lecture notes and their distribution to my Stony Brook students was my inability to find textbooks I could, with a clear conscience, use for teaching. First, the good books I could find[4] did not quite match my class audiences, which included not only theory-oriented but also experiment-oriented students, some PhD candidates from other departments, some college graduates with substandard undergraduate backgrounds, and a few advanced undergraduates. Second, for the rigid time restrictions imposed on the core physics courses, most available textbooks are way too long, and using them would mean hopping from one topic to another, picking up a chapter here and a section there, at a high risk of losing the necessary background material and logical connections between course components – and students' interest with them.[5].

The main objective of these materials is to make students familiar with the basic notions and ideas of physics that, in my humble view, have to be necessary parts of *every* professional physicist's education – hence the title of the series. As a backside of such a limited goal, I believe that my texts may be used by advanced undergraduate physics students as well. Moreover, I hope that some parts of the series may be useful for graduate students of other disciplines, including astronomy, chemistry, electrical, computer, and electronic engineering, mechanical engineering, and material science.

Of course, real mastering a new concept or method is impossible without the skills to apply it to particular situations. This is why in this series, a heavy accent is made on problem solution. In particular, the introduction of every new theoretical technique is always accompanied by an application

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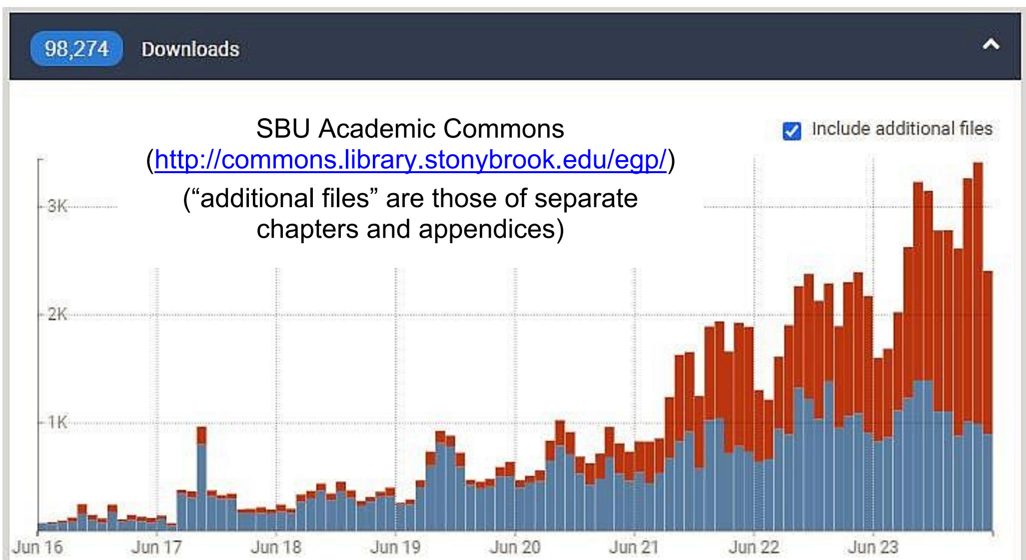
[4] In addition to the textbooks listed in *References*, several high-quality graduate-level teaching materials are now freely available online, including (but not limited to) R. Fitzpatrick's *Classical Electromagnetism* (farside.ph.utexas.edu/teaching/jk1/Electromagnetism.pdf), M. Fowler's *Graduate Quantum Mechanics Lectures* (http://galileo.phys.virginia.edu/classes/751.mf1i.fall02/home.html), B. Simons' lecture notes on *Advanced Quantum Mechanics* (www.tcm.phy.cam.ac.uk/~bds10/aqp.html), and D. Tong's lecture notes on several topics (www.damtp.cam.ac.uk/user/tong/teaching.html).

[5] At the same time, many graduate-level textbooks lack even brief discussions of several traditional and modern topics that I deem essential. To list just a few: statics and dynamics of elastic and fluid continua, basic notions of physical kinetics, turbulence and deterministic chaos, physics of reversible and quantum computation, energy relaxation and dephasing of open quantum systems, the van der Pol method (a.k.a. the Rotating-Wave Approximation, RWA) in classical and quantum mechanics, electrons and holes in semiconductors, the weak-potential and tight-binding approximations in the energy band theory, optical fiber electrodynamics, macroscopic quantum effects in Bose-Einstein condensates, Bloch oscillations and Landau-Zener tunneling, cavity QED, and the Density Functional Theory (DFT). All these topics are discussed, if only concisely, in these notes.

example or two. Additional exercise problems are listed at the end of each chapter of the lecture notes; they may be used by instructors for homework assignments. Individual readers are strongly encouraged to solve as many of these problems as possible.[6]

Supplements with detailed model solutions of the exercise problems are also available online – for the link, see the *Short Preface* above. Sets of shorter problems suitable for tests, also with model solutions, are available for university instructors from the author – also free of charge, but in return for a signed commitment to avoid their unlimited distribution.[7]

A preliminary ("beta") version of this series was placed into open online access in early 2014. Since that time, its popularity has been steadily growing – possibly, facilitated by my continuous editing of the material and adding more problems. As a result, during the 2023/24 academic year, readers from 130+ countries all over the globe downloaded more than 30,000 files of these materials from just the basic website of the series – see the graphics below.[8] Let me hope that this "final"[9] version of *Essential Graduate Physics* would earn an even broader readership.



[6] The problems that require either longer calculations or more creative approaches (or both) are marked by asterisks.

[7] *Approximate* contents of the request (to be sent to *klikharev@gmail.com* from a valid university address):

Dear Dr. Likharev,

I plan to use your lecture notes and problems of the *Essential Graduate Physics* series, part ⟨select: CM, EM, QM, SM⟩, in my course ⟨title⟩ during ⟨semester, year⟩ in ⟨department, university⟩. I would appreciate your sending me the *Test Problems with Model Solutions* for that part. I will avoid unlimited distribution of these problems, in particular their posting on externally searchable websites. If I distribute the problems among my students, I will ask them to adhere to the same restraint. I will let you know of any significant typos/deficiencies I may find.

Sincerely, ⟨full name, university position, work phone number⟩

[8] A very early (and, admittedly, rather buggy) version of these materials was published in 2017-2019 by IOP as eight volumes under the general title *Essential Advanced Physics*. Since then, the series has been significantly extended and much better corrected and edited.

[9] My current plans are to continue its improvement, though probably at a slower pace than before.

2. Disclaimer

Since these materials are available online free of charge, and that their author is not getting any royalties from the sale of their paperback copies, it is hard to imagine somebody blaming him for deceiving "customers" for his commercial gain. Still, I would like to go a little bit beyond the usual litigation-avoiding statements,[10] and offer a word of caution to potential readers, to preempt their possible disappointment.

(i) These are NOT courses of theoretical physics in the contemporary sense of this term. Though much of the included material is very similar to that in older textbooks on "theoretical physics" (most notably, in the famous series by L. Landau and E. Lifshitz), my focus is on the basic concepts and ideas of physics and their most important applications, rather than on sophisticated theoretical techniques. Indeed, the set of theoretical methods discussed in the notes is limited to the minimum needed for quantitative formulations of the key notions of physics and applying them to baseline system models, as exemplified the examples and problems included in the series. Moreover, because of the time/space limits, I have not been able to cover some important fields of theoretical physics – most regretfully, general relativity,[11] and also quantum field theory (beyond the basic quantum electrodynamics discussed in QM Chapter 9). As a result, if you want to work in modern theoretical physics research, you need to know much more than what this series teaches.

(ii) My lecture notes are NOT textbooks – at least not the usual ones. A usual textbook tries (though most commonly fails) to cover virtually all aspects of the addressed field. As a result, it is typically way too long to be fully read and understood by students during the time allocated for the corresponding course. In contrast, my lecture notes are quite succinct – about 200 pages per semester, enabling their thorough reading. This briefness is partly compensated by a large number of problems, with some of them devoted to additional topics, so their solutions may serve as extensions of the lecture notes. I also give extensive further reading recommendations on the topics I had no time/space to cover.

To summarize again, what my materials (including the problem solutions) really ARE is a description of the physics' hard core that has to be mastered by *everybody* who wants to work in the field – whether it is in theory or experiment, fundamentals or applications.

3. Confessions

First, I have to confess that besides giving an introduction to the core of physics, my course series also had another goal: to convey my own enchantment by the unparalleled beauty of the basic concepts and ideas of this science, and the remarkable logic of their fusion into a wonderful single construct, which is only conditionally partitioned by the subject boundaries. (Hence so many cross-references between the parts.)

One more confession: I know very well that my texts are far from being perfect. While I have made a major effort to organize the material so that the students could readily follow and enjoy the

[10] Yes, Virginia, these notes represent only my personal opinions, not necessarily those of the Department of Physics and Astronomy of Stony Brook University, the SBU at large, the SUNY system as a whole, the Empire State of New York, the US federal agencies and private companies that funded my group's research, etc. No, dear, I cannot be held responsible for any harm, either bodily or mental, their reading may (?) cause.

[11] For an introduction to that subject, I can recommend either its review by S. Carroll, *Spacetime and Geometry*, Addison-Wesley, 2003, or a longer text by A. Zee, *Einstein Gravity in a Nutshell*, Princeton U. Press, 2013.

narrative, it is more than possible that, at least in some instances, I have not quite reached this goal. In addition, it is almost certain that despite all my efforts and the great help from my colleagues, students, teaching assistants, and some readers, listed below, not all typos (and possibly, even real errors) have been weeded out.

This is why all (however candid) remarks and suggestions by the readers would be highly appreciated; they may be sent to *klikharev@gmail.com*. All significant contributions will be gratefully acknowledged in future edits.

4. Acknowledgments

I am extremely grateful to my faculty colleagues and other readers who commented on certain sections of the notes; here is their list (in the alphabetic order):[12]

A. Abanov, P. Allen, D. Averin, S. Berkovich, P.-T. de Boer, M. Fernandez-Serra,
R. F. Hernandez, P. Johnson, T. Konstantinova, A. Korotkov, V. Semenov, F. Sheldon,
S. Sridharan, E. Tikhonov, O. Tikhonova, E. Voronina, X. Wang, T.-C. Wei.[13]

The Department of Physics and Astronomy of Stony Brook University was very responsive to my kind requests for certain time ordering of my teaching assignments, which was beneficial for note writing and editing. The department, and the university as a whole, also provided a very friendly general environment for my work there for almost three decades.

A large part of my scientific background and research experience reflected in these materials came from my education and work in the Department of Physics of Moscow State University.

And last but not least, I am very grateful to my wife Lioudmila for several good bits of advice on aesthetic aspects of note typesetting, and more importantly, for all her love, care, and patience – without them, this writing project would be impossible.

K. K. Likharev
https://you.stonybrook.edu/likharev/
June 2024

[12] I am very sorry that I have not kept proper records from the beginning of my lectures at Stony Brook, so I cannot list all the numerous students and TAs who had kindly attracted my attention to typos in earlier versions of these notes. Needless to say, I am very grateful to them all as well.

[13] Obviously, these kind people are not responsible for the remaining deficiencies.

Notation

Abbreviations

Eq. any displayed formula

Fig. figure

Sec. section

c.c. complex conjugate

h.c. Hermitian conjugate

Fonts

F, $\mathcal{F}$ scalar variables[14,15]

$\mathbf{F}$, $\boldsymbol{\mathcal{F}}$ vector variables

$\hat{F}, \hat{\mathcal{F}}$ scalar operators

$\hat{\mathbf{F}}, \hat{\boldsymbol{\mathcal{F}}}$ vector operators

F matrix

$F_{jj'}$ matrix element

Symbols

$\dot{}$ time differentiation (d/dt)

∇ spatial differentiation vector (*del*)

$\approx$ approximately equal to

$\sim$ of the same order as

$\propto$ proportional to

$\equiv$ equal to by definition (or evidently)

$\cdot$ scalar ("dot-") product

$\times$ vector ("cross-") product[16]

$\overline{\quad}$ time averaging

$\langle \ \rangle$ statistical averaging

[,] commutator

{ , } anticommutator[17]

n unit vector

Subject parts of the series

CM: Classical Mechanics

EM: Classical Electrodynamics

QM: Quantum Mechanics

SM: Statistical Mechanics

Appendices

MA: Selected Mathematical Formulas

UCA: Selected Units and Constants

Frames

In the lecture notes, the most general and/or important formulas are highlighted with blue frames and short titles on the margins.

Numbering

Chapter numbers are dropped in all references to formulas, figures, footnotes, and problems within the same chapter.

Prime signs

In most cases, the prime signs (′, ″, etc.) are used to distinguish similar variables or indices (such as j and j' in the matrix element above), rather than to denote derivatives.

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14 The same letter, if typeset in different fonts, typically denotes different variables.

15 In the plain text, Italic fonts are used for emphasis – in particular, of the new terms at their first usage.

16 On a few occasions, the cross sign is used to emphasize the usual multiplication of scalars.

17 Following a venerable tradition, the curly brackets ("braces") are also used to enclose the exponential function arguments if they are typeset inline, e.g., $\exp\{-x/\delta\}$.

General Table of Contents

(besides this *Front Matter*)

QM: Quantum Mechanics	*Pages*	*Exercise Problems*
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Chapter 3. Higher Dimensionality Effects	64	50
Chapter 4. Bra-ket Formalism	52	36
Chapter 5. Some Exactly Solvable Problems	48	55
Chapter 6. Perturbative Approaches	36	31
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SM: Statistical Mechanics	*Pages*	*Exercise Problems*
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Chapter 5. Fluctuations	44	30
Chapter 6. Elements of Kinetics	38	18
Lecture Notes and Problems Total	224	156
Supplement: **Exercise Problems with Model Solutions**	252	156
Additional file (available for course instructors upon request):	*Pages*	*Problems*
Test Problems with Model Solutions	25	26

Appendices	*Pages*
MA: Selected Mathematical Formulas	16
UCA: Selected Units and Constants	4

References	*Pages*
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